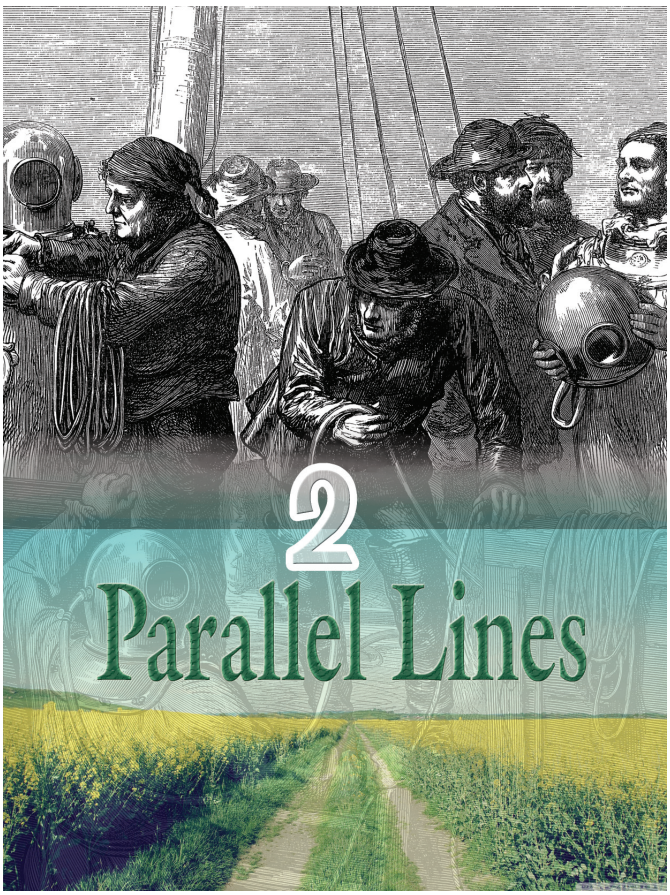
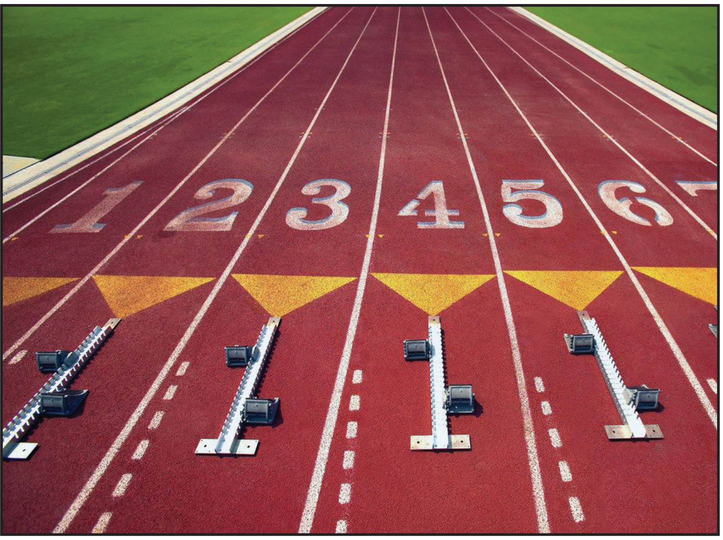
2

Parallel Lines

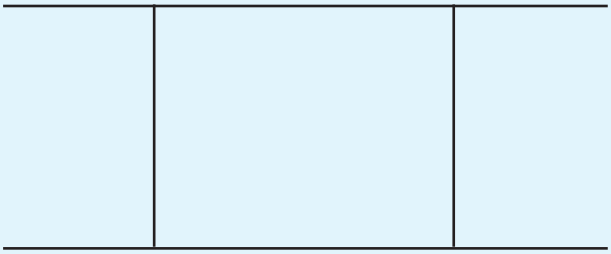


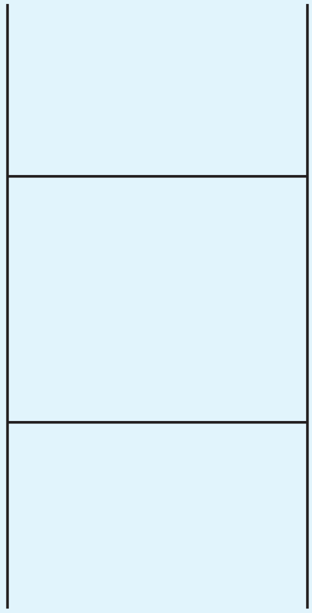


Parallel Lines

We get a line by joining any two points; on the other hand, do any two lines meet at a point?

What about the lines got by extending a pair of opposite sides of a rectangle?

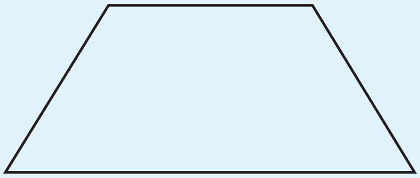




Do they meet however much we extend them?

Why?

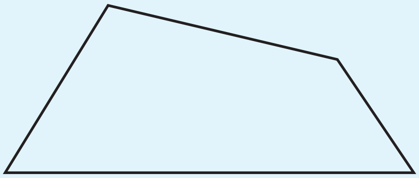
Look at this quadrilateral:



If we extend the top and bottom sides, do they meet?

What about the left and right sides?

What if we draw a quadrilateral like this?



Does any pair of opposite sides meet if we extend them?

Why?

Lines which are at the same distance everywhere, and do not meet anywhere, are called parallel.

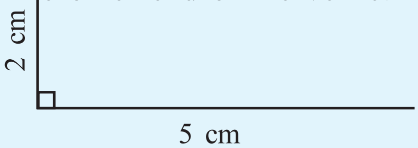
Same distance

Don't you know how to draw a rectangle?

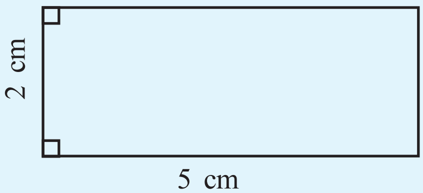
How do we draw a rectangle of length 5 centimetre and breadth 3 centimetre?

There are several ways, right?

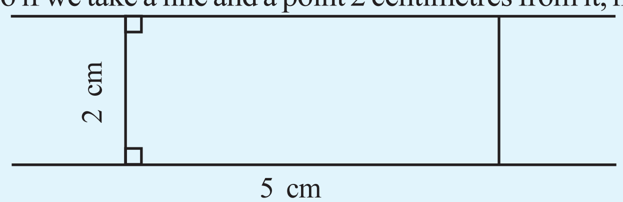
First draw a horizontal line 5 centimetre long and draw a vertical line at one end, 2 centimetre high:



Next at the other end of the vertical line, draw a perpendicular 5 centimetres long. Joining the other end of this perpendicular to the end of the first line makes our rectangle:



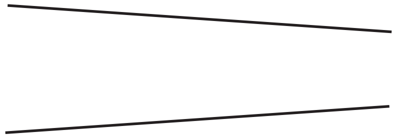
Now extend the top and bottom sides of this rectangle; we get a pair of parallel lines:



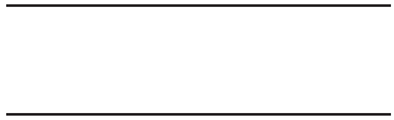
So if we take a line and a point 2 centimetres from it, how

Distance

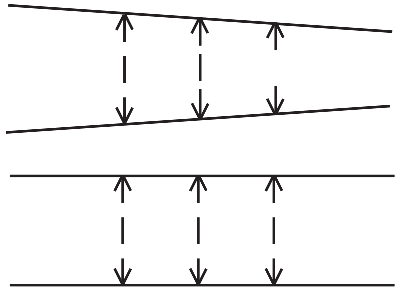
Would these lines meet, if extended?



How about these?

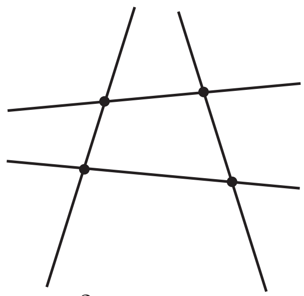


Look at the distance between the lines in both the figures:



What can we say about the distance between two parallel lines?

In GeoGebra, draw a quadrilateral. Use the **Line through two points** tool to extend the sides.



Do they meet?

Use move tool to drag the corners of the quadrilateral. When do the sides fail to meet?

Perpendicular and parallel

See the figure:

Look at the perpendicular lines to the horizontal.

Are they parallel?

Now see this:

A perpendicular is drawn to the horizontal line and then a perpendicular to the perpendicular is drawn.

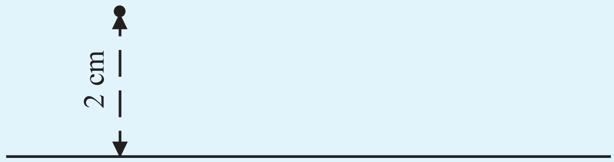
Are the horizontal lines parallel?

There are tools in GeoGebra to draw parallel and perpendiculars to a line. First draw a line and mark a point on it. Select the **Perpendicular line** tool, and click on the line and point, to get a perpendicular to the line through the point. Such a perpendicular can be drawn through a point not on the line also. Try drawing a perpendicular to this perpendicular.

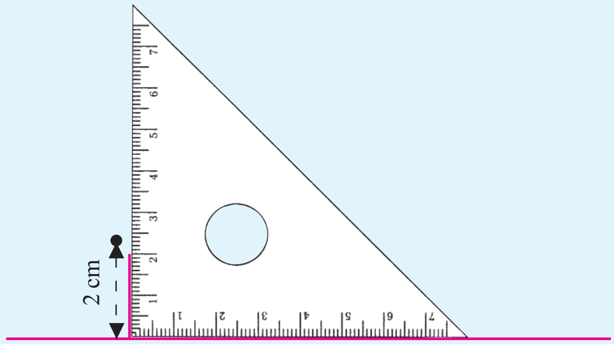
To draw a line parallel to another, the **Parallel line** tool is used. Mark a point not on the line. Select this tool and click on the line and the point, to get the parallel through the point. Drag the point using the **Move** tool. What happens if the point is on the line?

do we a draw a parallel to the line through the point?

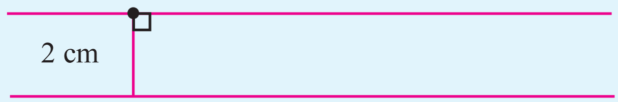
First draw the perpendicular to the line through the point:



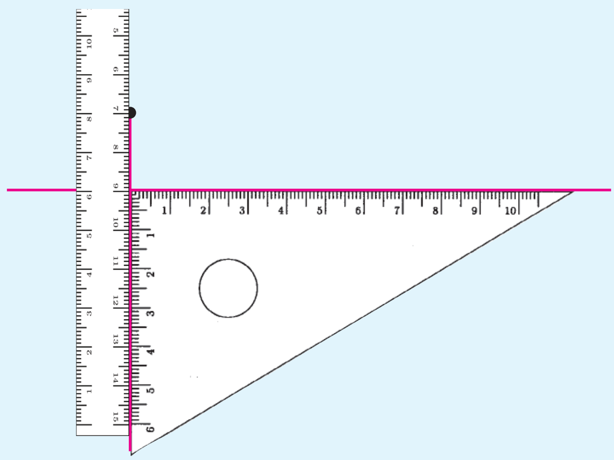
Then draw the perpendicular to this perpendicular through



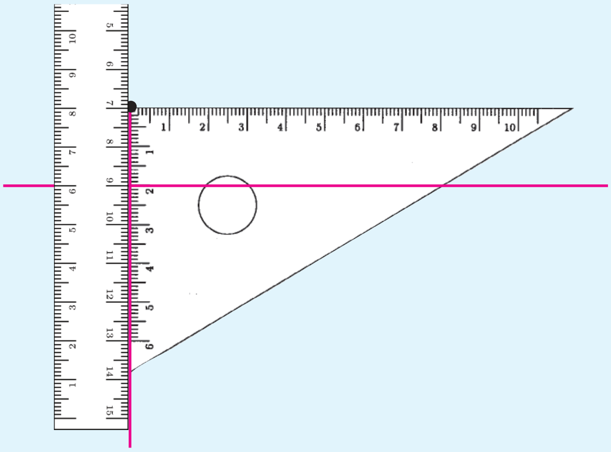
the point:



Instead of actually drawing the first perpendicular, we can use a ruler:



Now by shifting the set square upward and putting its square corner at the point, we can draw the parallel:



What if the point is below the line?

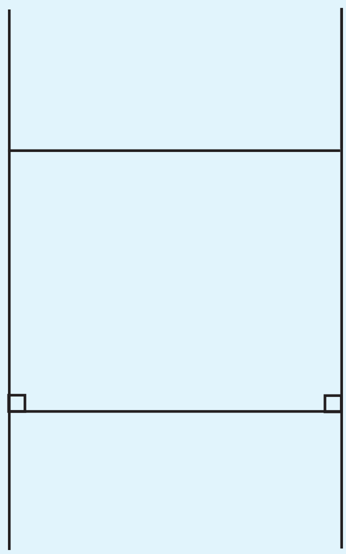
What did we see here?

Through any point not on a line, we can draw a parallel line.

How many parallels can be drawn through a point not on a line?

Same direction

Opposite sides of a rectangle are parallel:

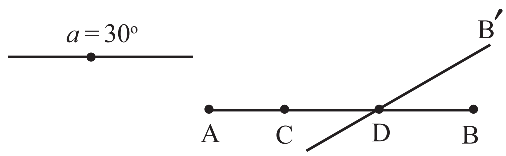


This can be said in a different manner: Two perpendiculars to the same line are parallel.

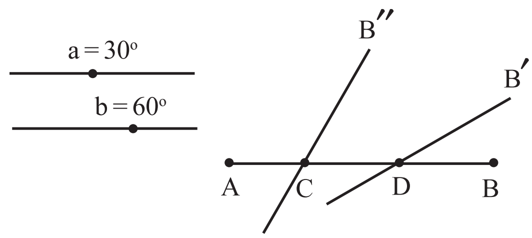
Draw line AB in GeoGebra and mark two other points C, D on it.

Now select the **Slider** tool and click on the GeoGebra screen. In the dialog box, select the **Angle** option by clicking on the small circle beside it. Against the **Name**, type `a`. Now click **Apply**.

Select the **Angle with given size tool** and click on B and D. In the dialog box, type `a` as **Angles** and click **OK**. Now we get a point B'. Join D and B' by a line.



Now make another slider named `b`. Select the **Angle** tool and click on B and C in that order. In the dialog box, type `b` as the angle and click **OK**. Join the new point B'' with C.

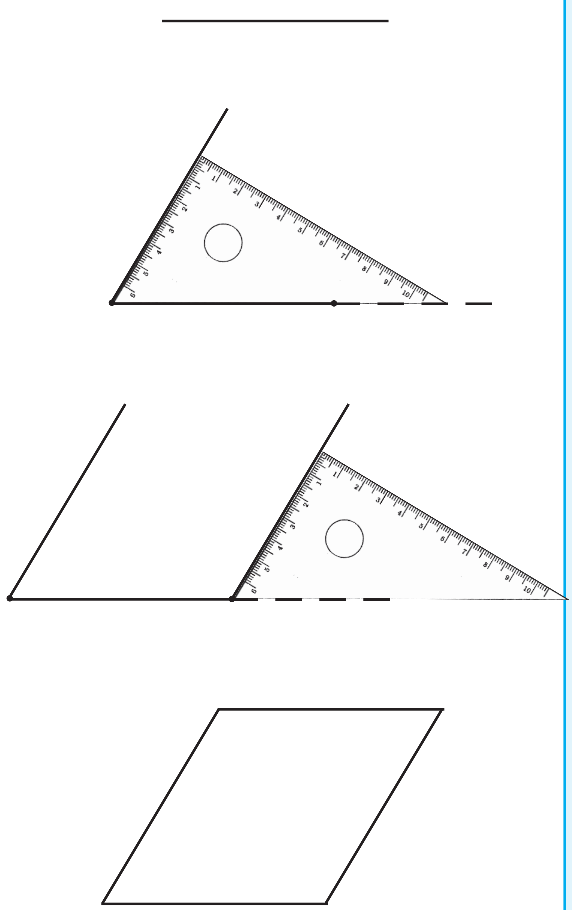


Use the **Move** tool to change the values of `a` and `b`. What happens to the lines? When do they fail to meet?

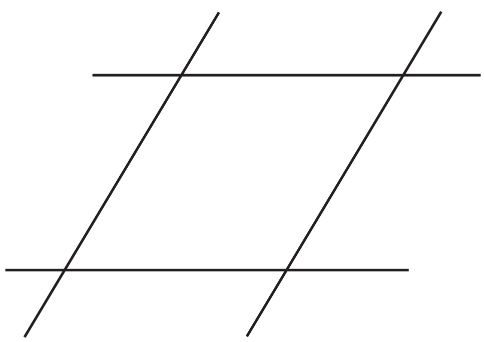
Try these with a single slider for the angles at C and D.

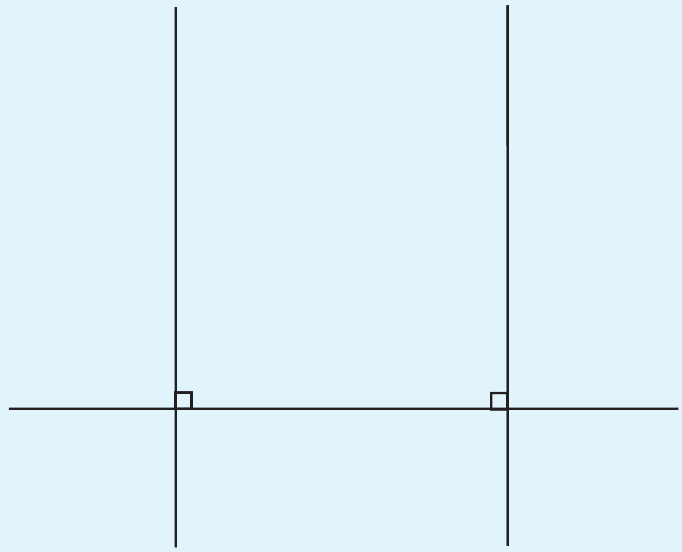
Not a square, but...

You know how to draw rectangles, using a set square. What if we use another corner, instead of the square one?

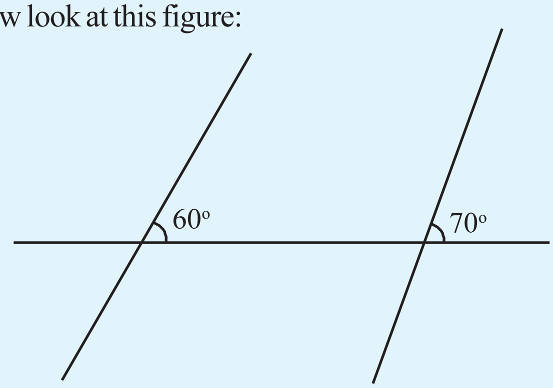


Does any pair of opposite sides meet, when extended?





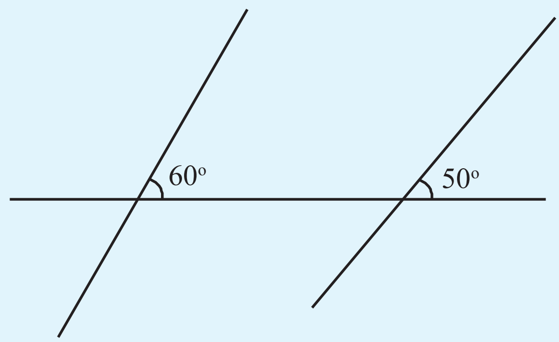
Now look at this figure:



Are the slanted lines parallel?

What would happen if these lines are extended upwards?

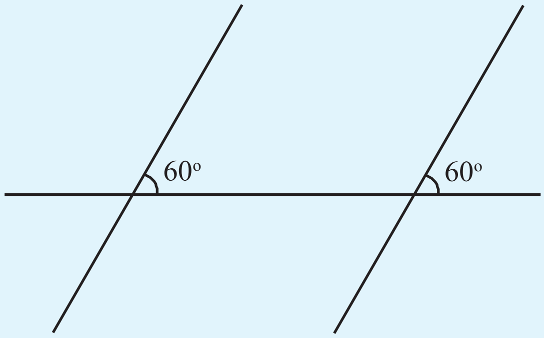
What if the lines are like this?



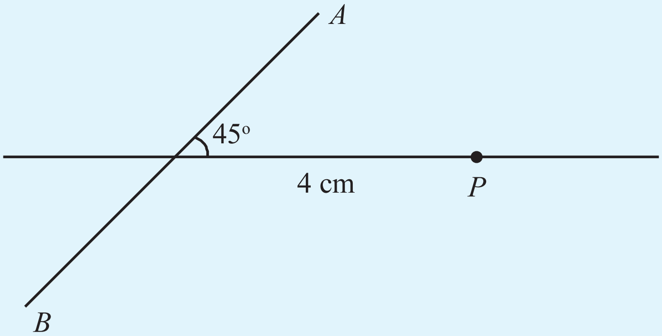
Would the lines meet if extended upwards?

Suppose we extend them downwards?

If the lines are not to meet either way, by how much should the right line be slanted?

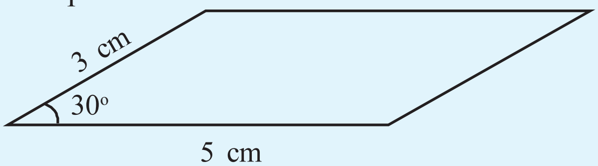


Now draw a figure like this in your notebook:



Is there a quick and easy way to draw a line parallel to *AB* through *P*?

In the quadrilateral shown below, both pairs of opposite sides are parallel:

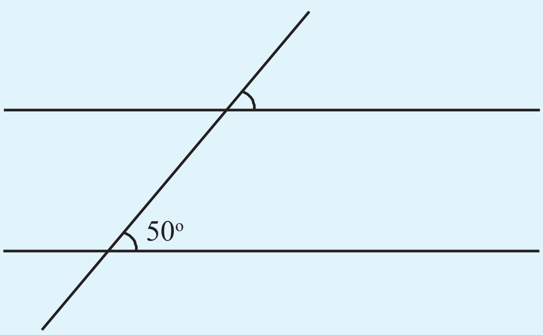


Can you draw this with the same lengths and angle?

A quadrilateral like this, with opposite sides parallel, is called a *parallelogram*.

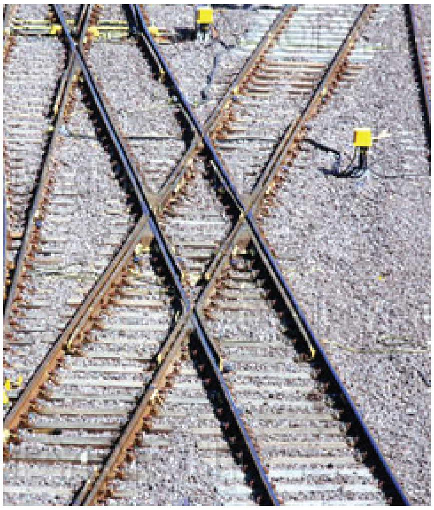
Parallels and angles

In the figure below, the top and bottom lines are parallel:



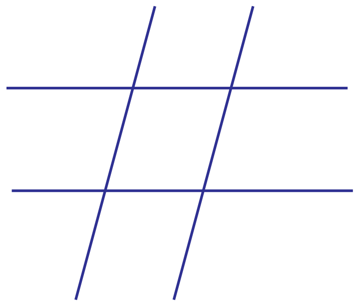
How much is the angle marked on the top?

When parallels cut

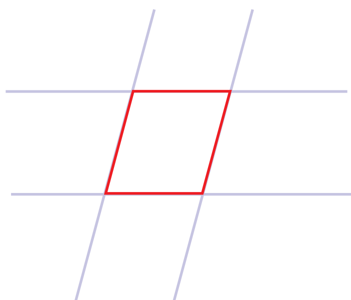


Draw a pair of parallel lines.

And another pair cutting across them.



Look at the figure made between them:

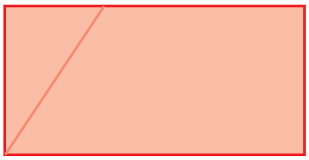


What is its name?

Rectangle and parallelogram

Cut out a cardboard rectangle:

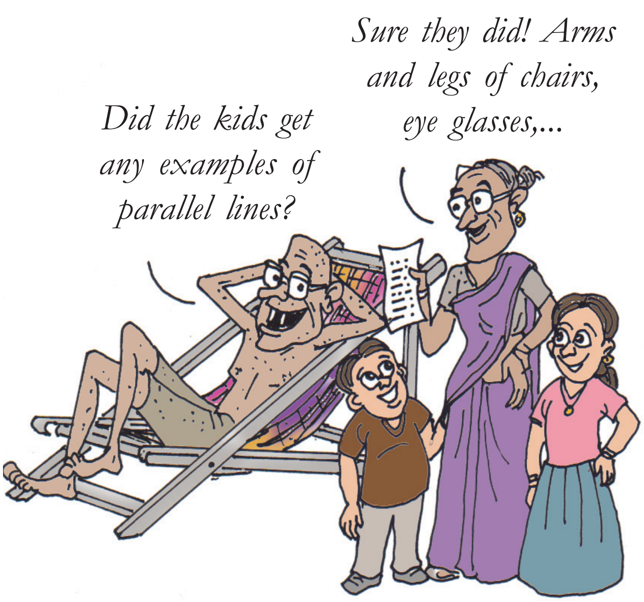
Now cut a triangle through the bottom corner as shown below:



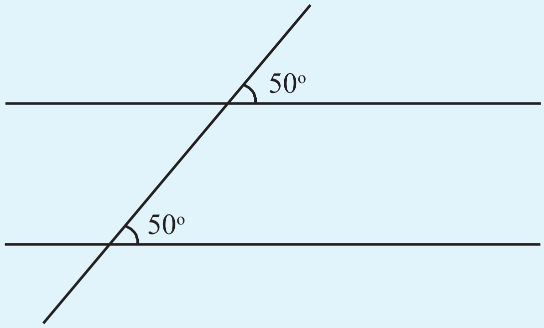
Place the triangle on the other side like this.

Is this a parallelogram?

Why?

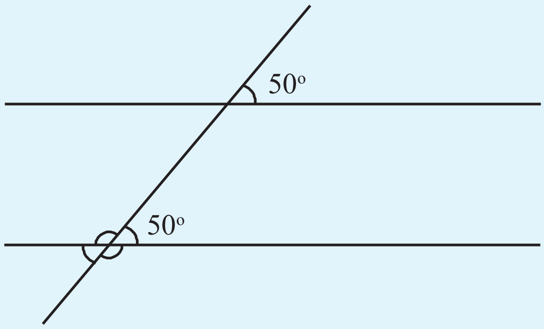


Parallel lines should have the same slant with any other line:

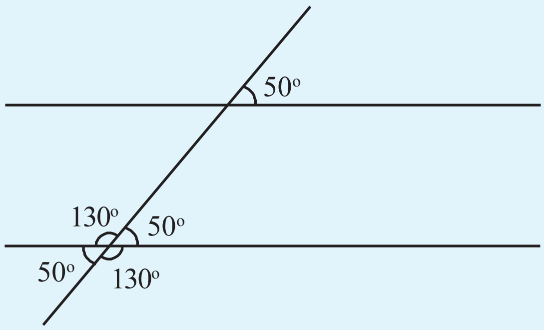


There are other angles here. Can you figure out those?

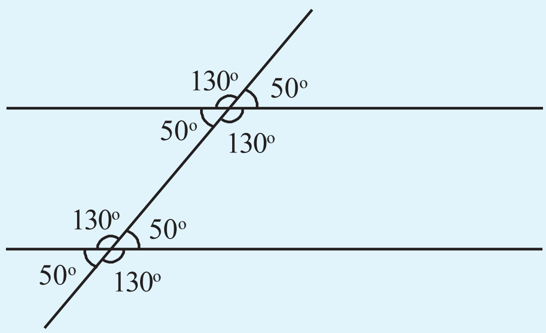
First look at the three angles below:



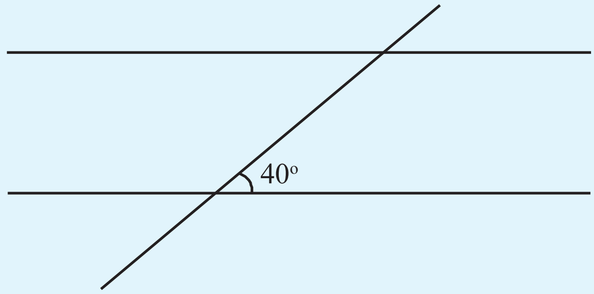
What are the relations between the four angles made by two lines cutting across each other?



Can't you find out the angles at the top also like this?



In the next figure also, the lines at the top and bottom are parallel:

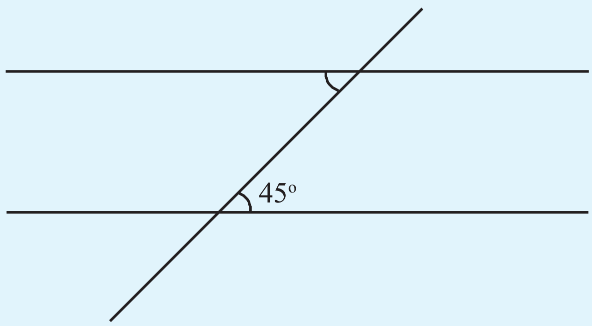


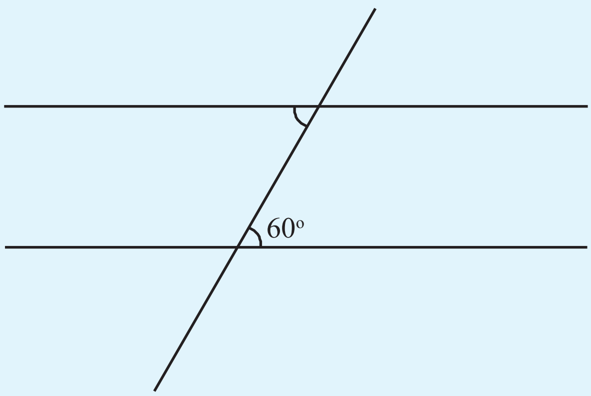
Write down the measures of the other seven angles in this figure.

What we have seen here can be written thus:

Parallel lines make equal angles with any other line.

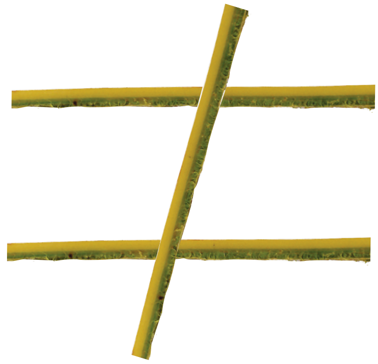
In the figures below, there is a pair of parallel lines and a third line cutting across them. In each figure, the measure of one angle is given and another angle is marked. Find out its measure and write it in the figure itself.



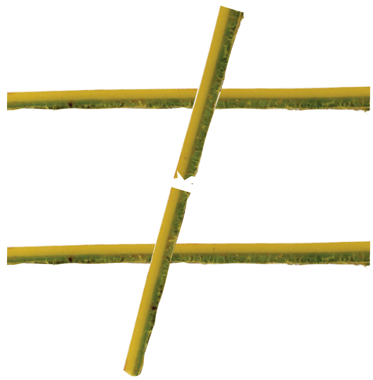


Unchanging shapes

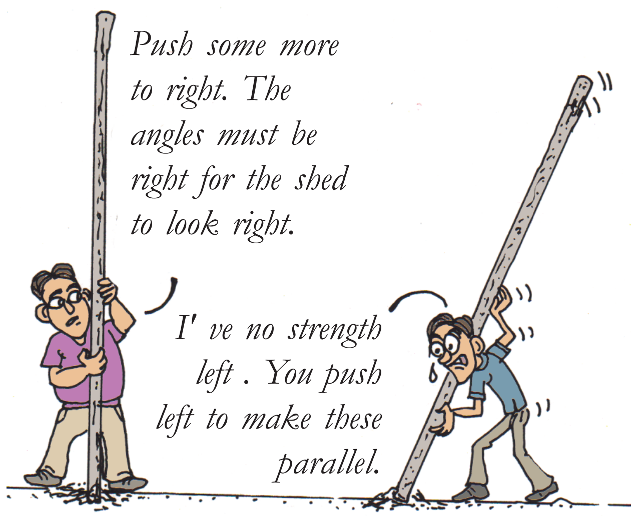
Place two straight and slender twigs parallel to each other. Place another one across and paste them together:



Now break at the middle to get two pieces:



Place one piece over the other. All these angles match, don't they?

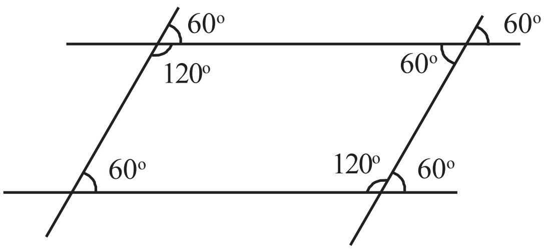


Angles of a parallelogram

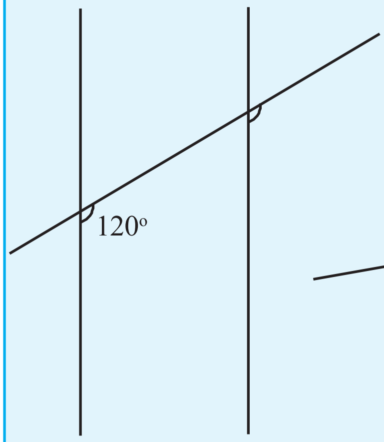
All angles in a rectangle are right angles.

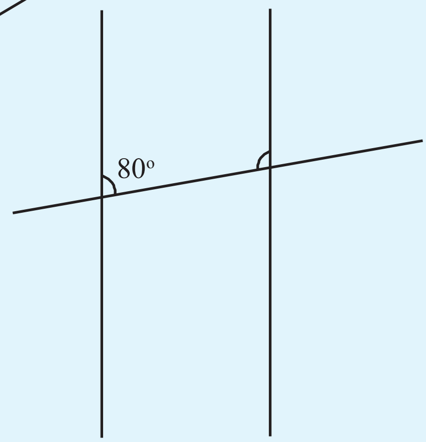
What about the angles in a parallelogram?

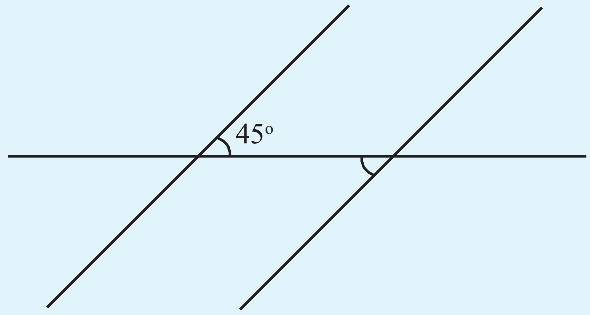
Can you find the angles in the first parallelogram?



Now find the angles in the second one.

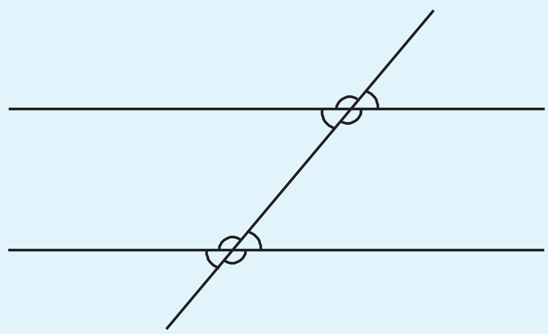






Matching angles

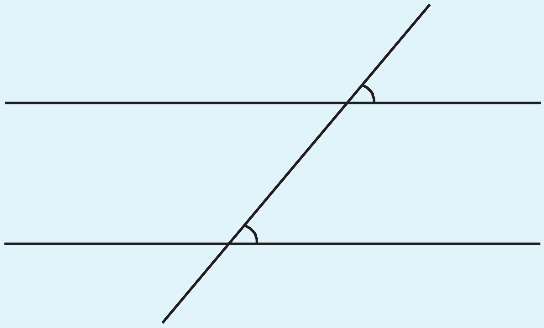
When a line cuts across a pair of parallel lines, eight angles are formed:



In the figure, the bottom line makes four angles, and the top line makes another four angles with the line cutting across them.

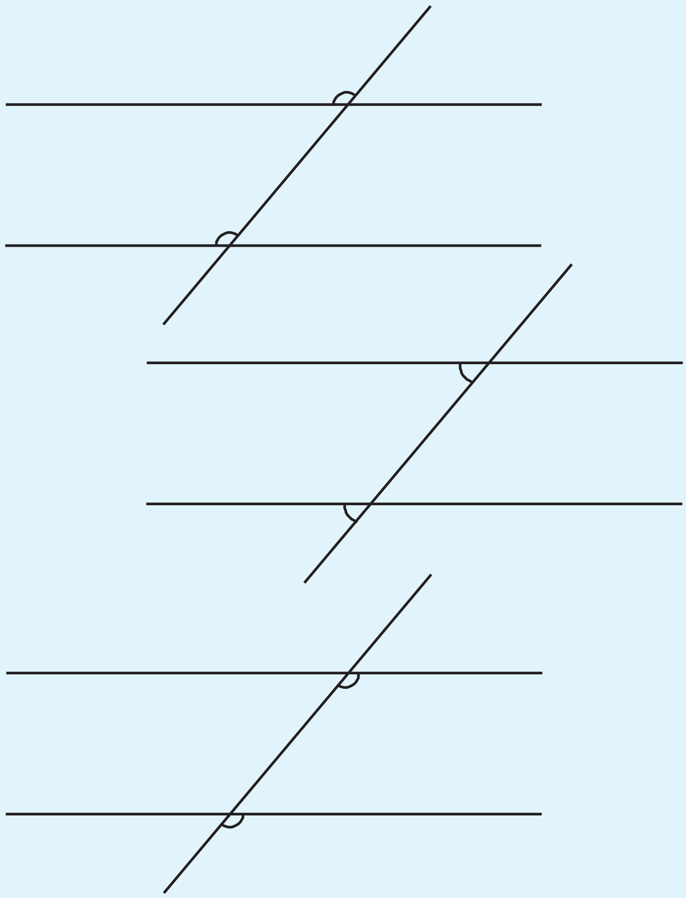
We can pair one angle at the bottom with one at the top in several different ways. Some such pairs are equal; others are supplementary (meaning their sum is 180°).

Let's look at the pairs of equal angles. For convenience, they are divided into two types. Look at the pair of angles marked in the figure below:



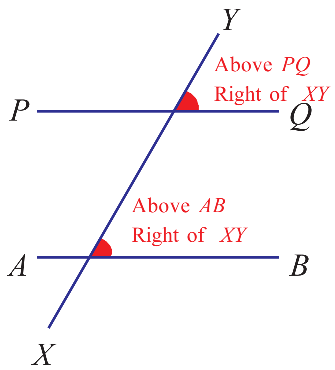
Of these, the angle at the bottom is on top of the horizontal line, and on the right of the slanted line; the angle at the top also is on top of its horizontal line, and on the right of the slanted line.

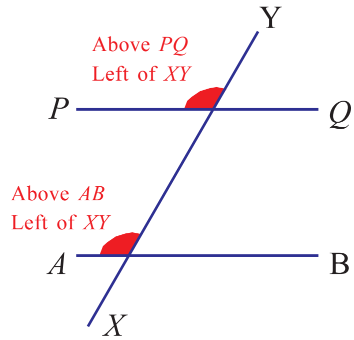
There are three more pairs of angles which are at similar positions at the bottom and top:

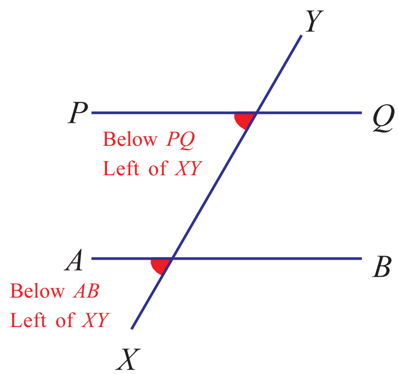


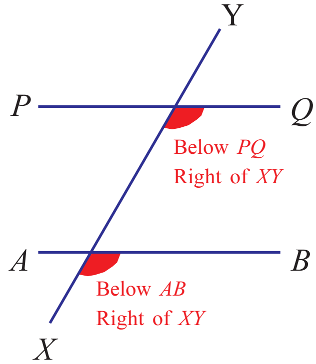
Angles in each such pairing, done according to similar positions, are called *corresponding angles*.

Corresponding angles



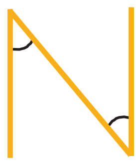






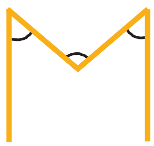
Alphabet angles

Draw the letter N as below:



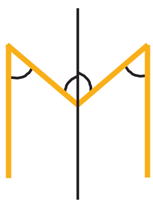
What is the relation between the two angles marked?

Now look at the letter M.

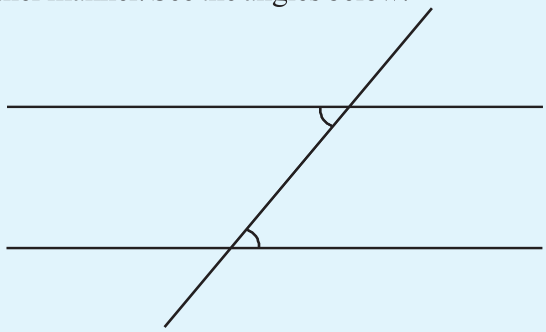


See any relation between the angles?

Draw a line down the middle and see:



Equal angles from the bottom and top can be paired in another manner. See the angles below:

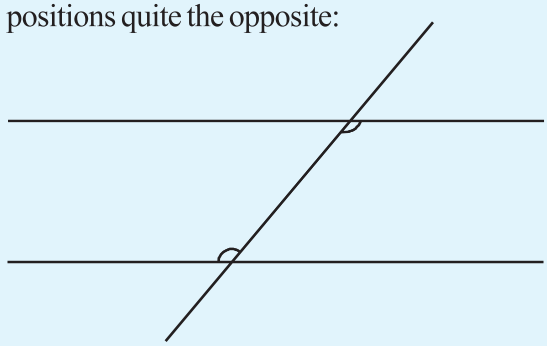


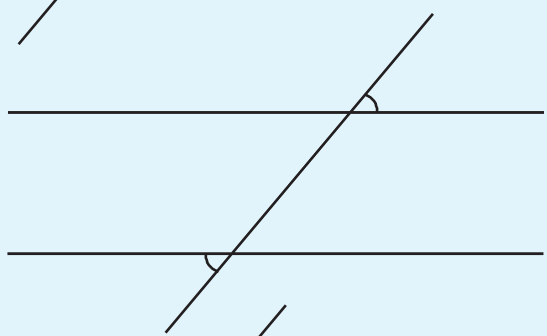
The bottom angle is on top of the horizontal line, and on the right of the slanted line

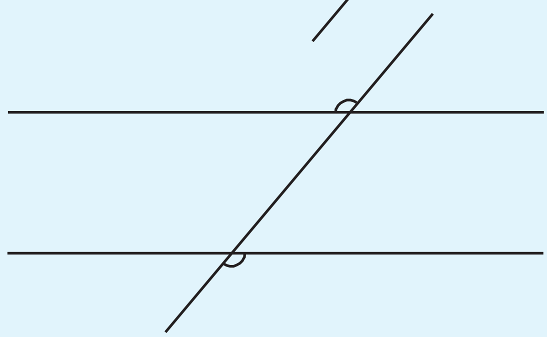
What about the top angle?

At the bottom of the horizontal line, and on the left of the slanted line.

We can pair the equal angles in three other ways, with the positions quite the opposite:

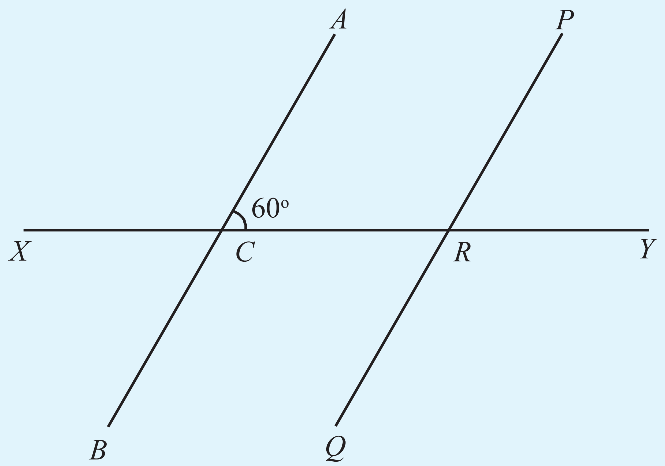






Angles in each such pairing, done with reverse positions, are called *alternate angles*.

In the figure below, the pair of parallel lines and the cutting line are all named. The measure of one angle is also given. Complete the tables below by writing the names and measures of all pairs of corresponding and alternate angles:



Corresponding angles	
Names	**Measure**
$\angle ACY$, $\angle PRY$	60°

Alternate angles	
Names	**Measure**
$\angle ACY$, $\angle QRX$	60°

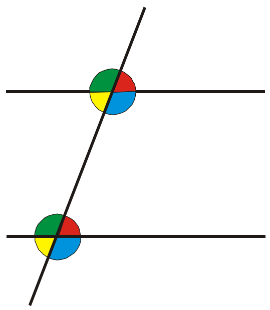
In short,

The angles formed by a line cutting across two parallel lines can be paired in several ways, choosing one angle of the four made with one line and one of the four made with the other. Of these, eight pairs have equal angles. Based on the positions with respect to the lines, angles in four such pairs are called corresponding and angles in the other four are called alternate.

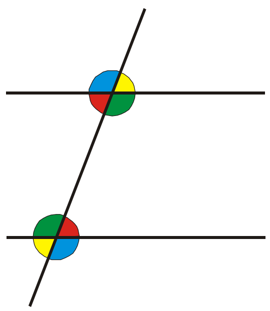
Corresponding and alternate

Look at the picture.

Pairs of corresponding angles are of the same colour.

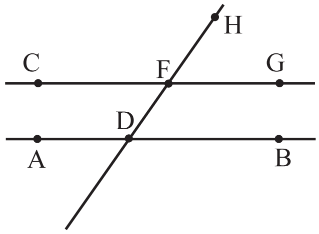


What about this picture?



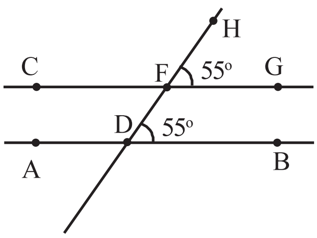
In GeoGebra draw a line AB and a parallel line through C.

Mark points D and F on these and join them. Mark two points G and H as in the figure.



Next select the **Angle** tool and click on G, F, H in that order and then on B, D, F.

Now we can see the measure of these angles.



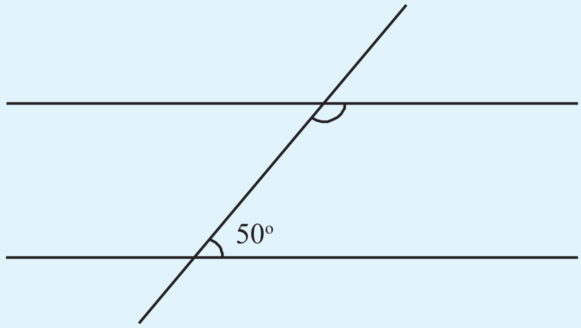
Use the **Move** tool to change the position of F.

Find the other angles at F and D like this.

We can also colour these angles. Right click on any angle and select **Object properties**. In the menu, click on **Color** and choose a colour. Choose the same colour for equal angles at the top and bottom.

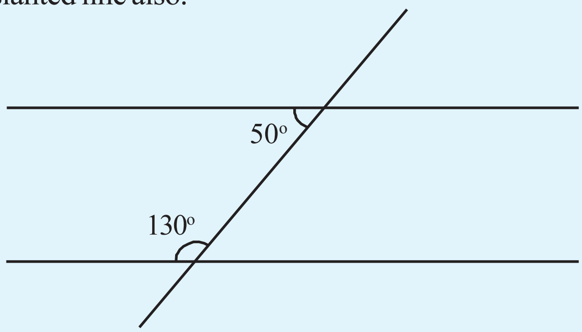
Supplementary angles

Let's have another look at a picture of a two parallel lines cut by a third line:



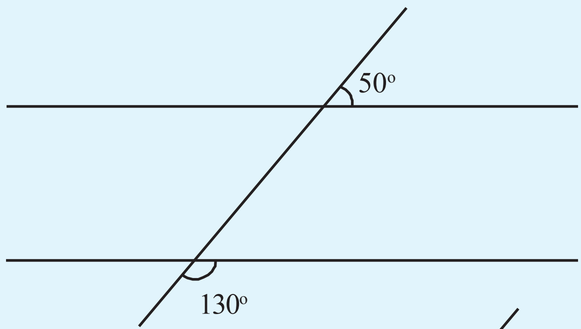
How much is the marked upper angle?

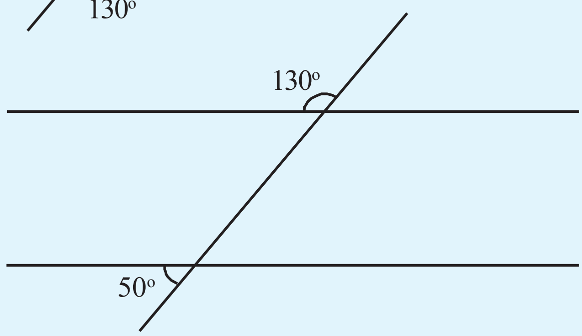
There is such a pair of supplementary angles on the left of the slanted line also.



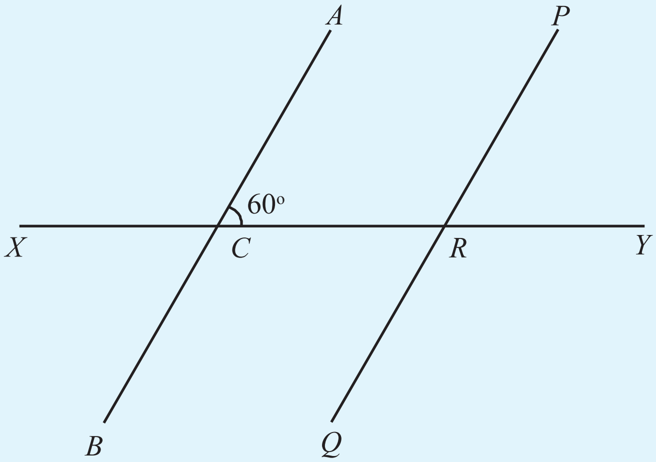
The angles in each of these two pairs are called *co-interior*.

There are also two pairs of *co-exterior* angles.





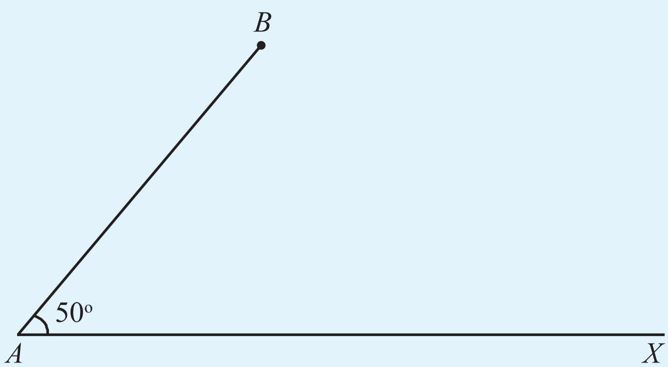
In the figure below, the lines *AB* and *PQ* are parallel and the line *XY* cuts them at *C* and *R*. Find all the pairs of co-interior and co-exterior angles and write down their names and measures.



Co-interior angles	**Co-exterior angles**

Parallel lines and triangles

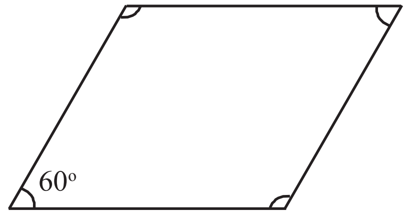
See this figure:



A line starting from *B* is to be drawn, parallel to *AX*.

Sums of angles

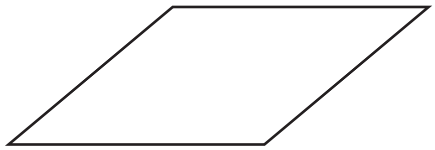
Look at the parallelogram:



Can you write the measures of the other three angles?

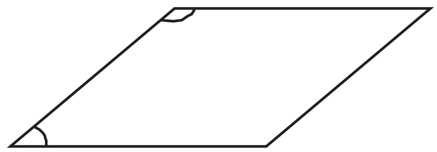
What is the sum of all the angles?

Now look at this parallelogram:

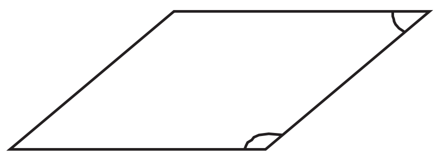


No angle is given.

But can't you say what the sum of the two angles on the left is?



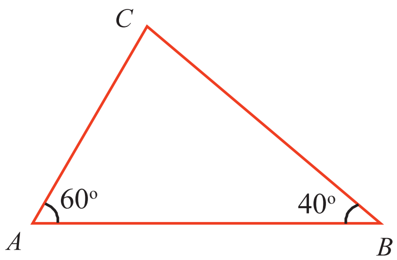
What about those on the right?



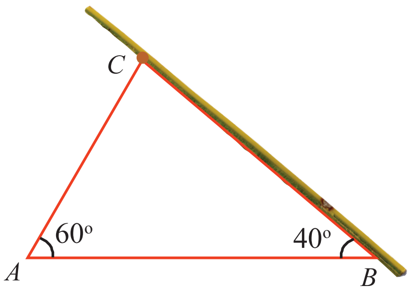
So, what is the sum of all four angles?

Triangle and parallel lines

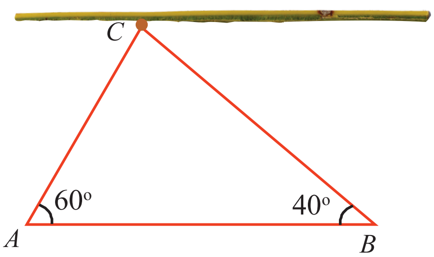
Draw a triangle like this in a piece of card-board.



Now place a long and thin stick along the side *BC* and stick a pin through it at *C*.



Rotate the stick upwards till it is parallel to *AB*.



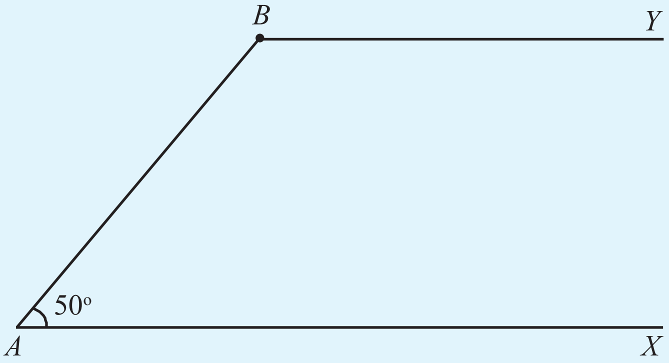
Now what angle does the stick make with *BC*?

And with *AC*?

So, how much is the angle at *C* in the triangle?

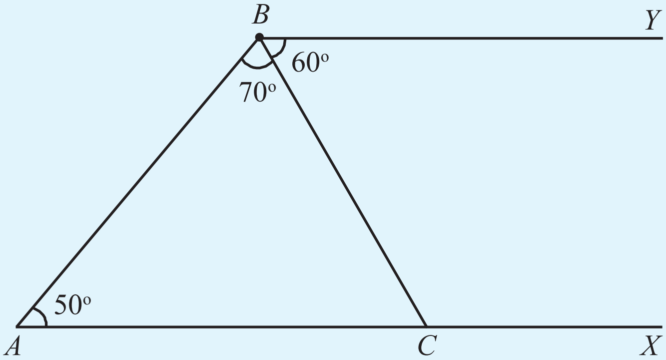
How do we do this?

The angles at *A* and *B* are co-interior, right?



Draw this in your notebook.

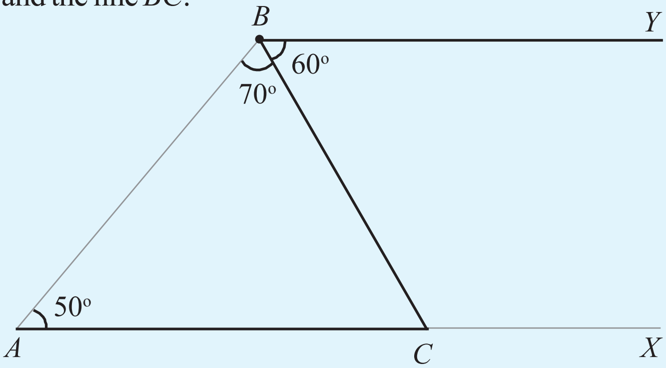
Next draw a slanted line through *B* in this figure. Let the angle with *AB* be 70°. This line is not parallel to *AX*. Let the point at which this line meets *AX* be named *C*:



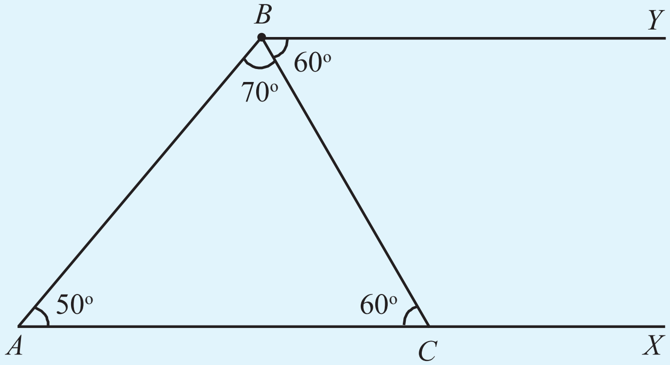
Now *ABC* is a triangle. And we know the angles at *A* and *B*.

How much is the angle at *C*?

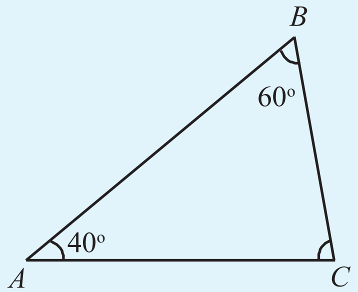
The lines *AC* and *BY* are parallel. Concentrate on these and the line *BC*:



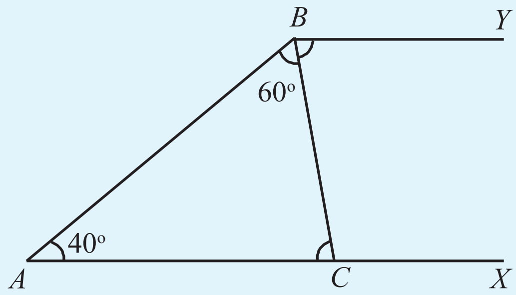
$\angle ACB$ and $\angle CBY$ are alternate angles:



Likewise, can you compute the angle at C in the triangle below?



How about extending AC and drawing a line from B parallel to it, like the first figure?



We have to calculate $\angle ACB$.

First, we note that it is equal to $\angle CBY$ (Why?).

To find $\angle CBY$, we need only know $\angle ABY$; and this angle together with $\angle A$ make a co-interior pair.

So,

$$\angle ABY = 180^\circ - 40^\circ = 140^\circ$$

From this we get

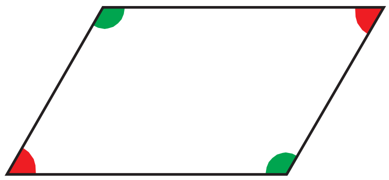
$$\angle CBY = 140^\circ - 60^\circ = 80^\circ$$

Thus we find

$$\angle ACB = \angle CBY = 80^0$$

Parallelogram and triangle

Look at this parallelogram:

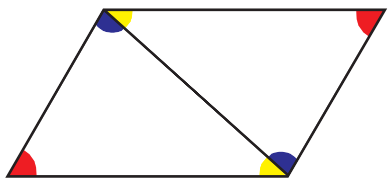


What is the relation between the angles marked red?

And the angles marked green?

And those marked in different colours?

Now join the opposite corners to make two triangles:



What is the relation between the angles marked blue?

And those marked yellow?

What is the sum of three angles of different colours?

What is the sum of the three angles of each triangle?

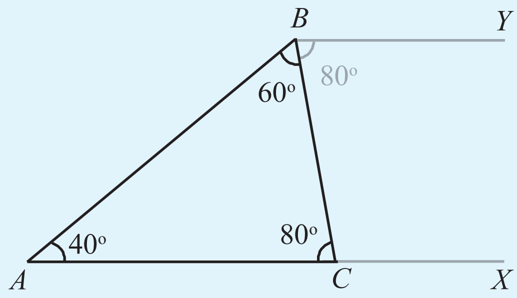
Theorem and proof

How do we conclude that the sum of the angles in any triangle is 180°? Is it enough if we draw several triangles, measure the angles and check the sum? How can we say for sure that for a triangle not among these, the sum is 180°?

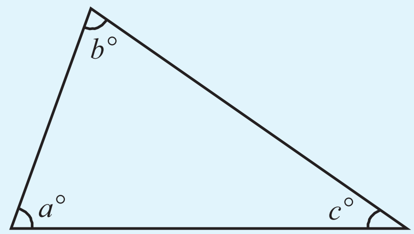
In any triangle, we can draw a line through one vertex, parallel to the opposite side. And then using the relations between angles made by parallel lines, we can see that the sum of the angles of a triangle is 180°.

By doing this, we achieve much.

- Even if we change the triangle, the arguments used do not change. So, the conclusion of these arguments also is true for the changed triangle.
- Properties of parallel lines can be easily recognized. But the fact that sum of the angles of a triangle is 180°, is not immediately obvious. This is an example of establishing complex ideas, starting from simple truths.
- When arguments are linked one after another using ideas of parallel lines, we not only get the theorem that sum of the angles of a triangle is 180°, but also see why it is so.

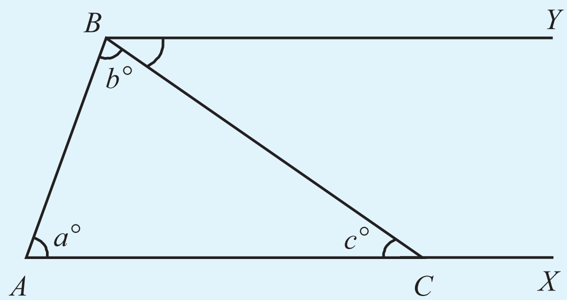


Now see this triangle:



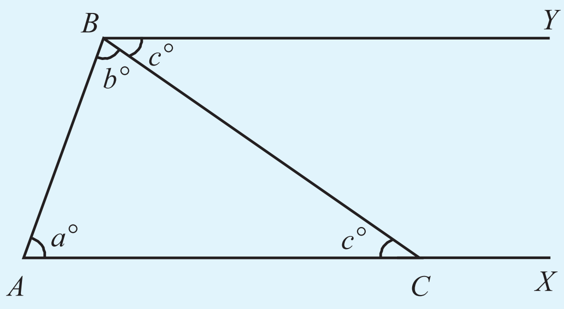
The measures of the angles are given by the letters a, b, c. What is the relation between them?

Let's draw parallel lines as before.



From this figure, we see that

$$\angle CBY = \angle ACB = c^\circ$$



From the figure above,

$$\angle A + \angle ABY = 180^\circ$$

That is,

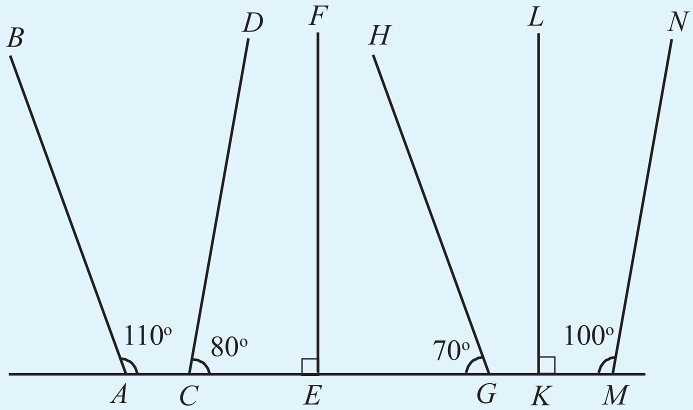
$$a + b + c = 180$$

What do we get from this?

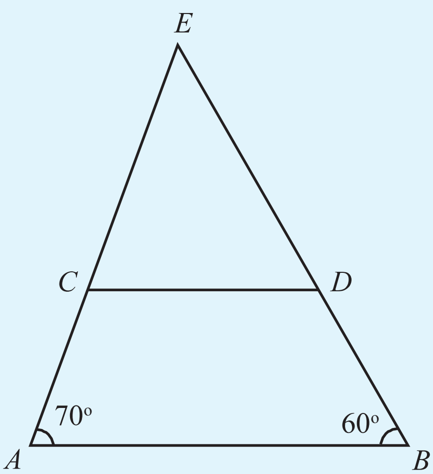
The sum of the angles of any triangle is 180°.

Let's do it!

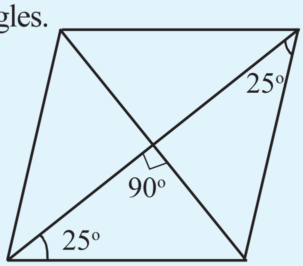
- Find out the pairs of parallel lines in the figure below:



- In the figure below, *AB* and *CD* are parallel. Compute all the angles in the figure.

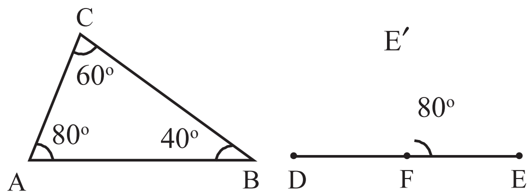


- In the figure below, a parallelogram is divided into four triangles by the diagonals. Calculate the angles of all these triangles.



Unchanging relation

In GeoGebra, use the polygon tool to draw triangle ABC. Using the **Angle** tool, we can get the measures of its angles.



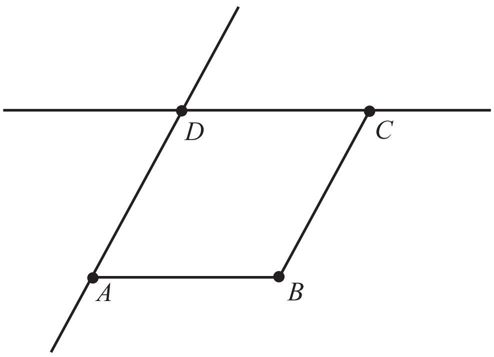
Now draw a line DE and mark a point F on it. Select the **Angle with given size** tool and click on E and F in that order. In the dialog box, type α as the angle and click **OK**. We get a new Point E′. With the same tool, click on E′ and F and type in β as the angles. We get a new point E″. Click on E″ and F and type γ as the angle to get another point E‴. Join FE′ and FE″. In this picture, we have ∠EFE' = ∠A ; ∠E′ FE″ = ∠B; ∠E″ FE‴ = ∠C. Give the same colours to equal angles.

Use the **Move** tool to change the triangle's angles. The angles in the figure on the right also change. What remains unchanged?

Drawing parallelograms

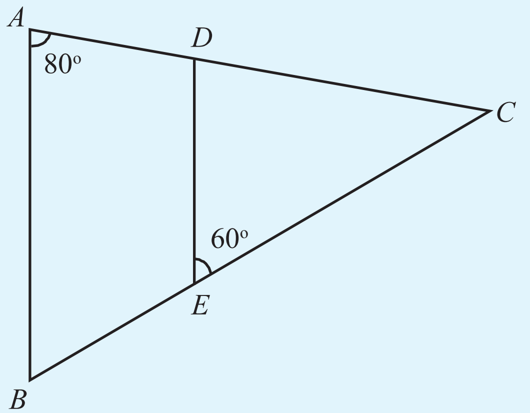
Let's draw a parallelogram using GeoGebra.

First draw two lines AB and BC. Use the **Parallel line** tool to draw the line through C parallel to AB and the line through A parallel to BC. Mark the point D where the lines meet. Use the **Polygon** tool to complete the parallelogram ABCD.



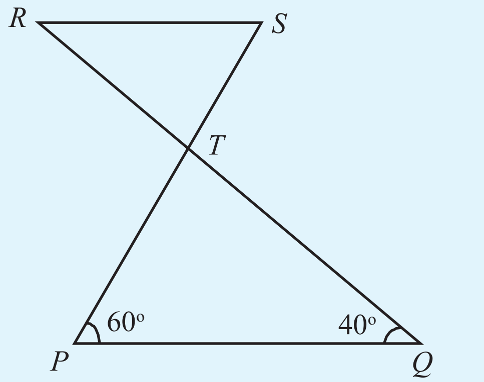
Lines sticking out may be deleted.

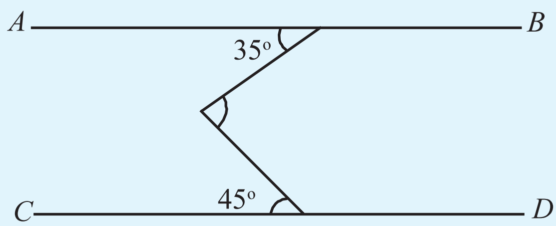
Now right click on AB and select **Trace on**. Do this for BC also. Select the **Move** tool, click within the parallelogram and drag upwards. What do you get?

- 

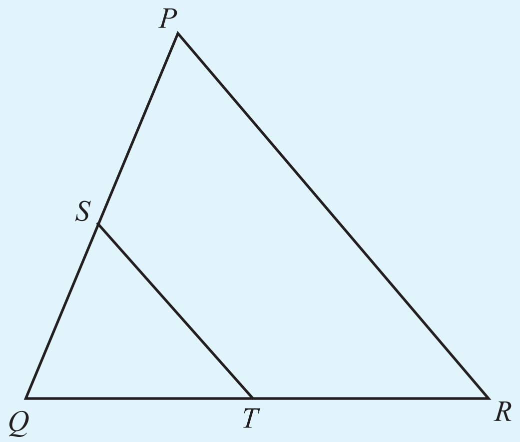
  In the figure above, *AB* and *DE* are parallel. Compute the angles of both triangles.

- In the figure below, *PQ* and *RS* are parallel. Calculate all other angles in the figure.

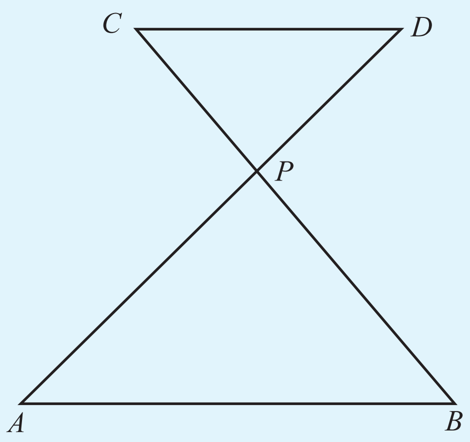


- 

  In the figure, *AB* and *CD* are parallel. Compute the third angle.

- 

  In the figure, *PR* and *ST* are parallel. Is there any relation among the angles of the two triangles?

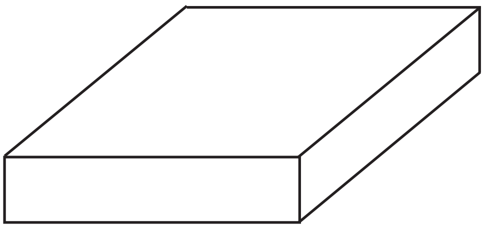
- 

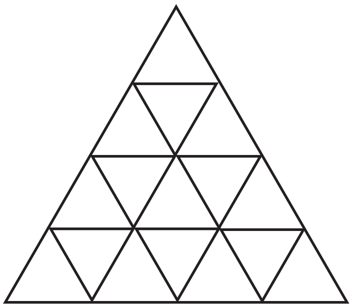
  In the figure above, *AB* and *CD* are parallel. What is the relation between the angles of the small and large triangles?

- Draw a line *AB* and a line *CD* parallel to it. Draw a line *EF* cutting across these lines at the points *M* and *N*. Measure and write down one of the angles so made. Calculate the other angles. Write down the pairs of corresponding angles, alternate angles, co- interior angles and co-exterior angles.

Drawing pictures

Try to draw these pictures using GeoGebra.





Use the **Regular Polygon** tool to draw the large triangle.

Looking back

Achievements	On my own	With teacher's help	Must improve
• Explaining parallel lines as lines which are a constant distance apart.			
• Explaining parallel lines in terms of perpendicularity and slant.			
• Drawing parallel lines using different methods and proving that they are parallel.			
• Explaining parallel lines using models.			
• Given one angle made by a line cutting across a pair of parallel lines, computing the other angles and justifying the computation.			
• Using computer to describe various properties of parallel lines.			
• Explaining the classification of pairs of corresponding, alternate, co-interior and co-exterior angles.			
• Proving that the sum of the angles of a triangle is 180°.			